Computational Fluid Dynamics

Module-No./Abbreviation	Credits	Workload	Term	Frequency	Duration
CE-WP05/CFD	6 CP	180 h	2nd Sem.	Summer term	1 Semester

Courses	Contact hours	Self-Study	Group Size:
Computational Fluid Dynamics	4 SWS (60 h)	120 h	No Restrictions

Prerequisites

Basic knowledge of: partial differential equations and their variational formulation, finite element methods, numerical methods for the solution of large linear and non-linear systems of equations

Learning goals / Competences

Students should become familiar with modern methods for the numerical solution of complicated flow problems. This includes: finite element and finite volume discretizations, a priori and a posteriori error analysis, adaptivity, advanced solution methods of the discrete problems including particular multigrid techniques.

After successfully completing the module, the students shall

- be familiar with the various equations describing fluid dynamics, in particular the Stokes equation, the compressible and incompressible Navier-Stokes equations and Euler, equations, as well as their scope and applicability,
- be able to select stable finite element discretizations for each type of equations and know its advantages, disadvantages, limitations and practical realization,
- know the convergence properties of the various methods and be able to describe when these convergence rates can be expected in practice,
- be able to formulate a posteriori error estimators and know how to use them to improve the efficiency of finite element methods.

Content

- 1) Modelization
  Velocity, Lagrangian / Eulerian representation; transport theorem, Cauchy theorem; conservation of mass, momentum and energy; compressible Navier-Stokes / Euler equations; nonstationary incompressible Navier-Stokes equations; stationary incompressible Navier-Stokes equations; Stokes equations; boundary conditions
- 2) Notations and auxiliary results
  Differential operators; Sobolev spaces and their norms; properties of Sobolev spaces; finite element partitions and their properties; finite element spaces; nodal bases
- 3) FE discretization of the Stokes equations, 1st attempt
  Stokes equations; variational formulation in {div u = 0}; non-existence of low-order finite element spaces in {div u = 0}; remedies
- 4) Mixed finite element discretization of the Stokes equations
  Mixed variational formulation; general structure of finite element approximation; an example of an instable low-order element; inf-sup condition; motivation via linear systems; catalogue of stable elements; error estimates; structure of discrete problem
- 5) Petrov-Galerkin stabilization
  Idea: consistent penalty term; general structure; catalogue of stabilizations; connection with bubble elements; structure of discrete problem; error estimates; choice of stabilization parameter

- 6) Non-conforming methods
  Idea; most important example; error estimates; local solenoidal bases
- 7) Streamline formulation
  Stream function; connection to bi-Laplacian; FE discretizations
- 8) Numerical solution of the discrete problems
  General structure and difficulty; Uzawa algorithm; improved version of Uzawa algorithm; multigrid; conjugate gradient variants
- 9) Adaptivity
  Aim of a posteriori error estimation and adaptivity; residual estimator; local Stokes problems; choice of refinement zones; refinement rules
- 10) FE discretization of the stationary incompressible Navier-Stokes equations
  variational problem; finite elements discretization; error estimates; streamline-diffusion stabilization; upwinding
- 11) Solution of the algebraic equations
  Newton iteration and its relatives; path tracking; non-linear Galerkin methods; multigrid
- 12) Adaptivity
  Error estimators; type of estimates; implementation
- 13) Finite element discretization of the instationary incompressible Navier-Stokes equations
  Variational problem; time-discretization; space discretization; numerical solution; projection schemes; characteristics; adaptivity
- 14) Space-time adaptivity
  Overview; residual a posteriori error estimator; time adaptivity; space adaptivity
- 15) Discretization of compressible and inviscid problems
  Systems in divergence form; finite volume schemes; construction of the partitions; relation to finite element methods; construction of numerical fluxes

Teaching methods / Language
Lecture (2h / week), Exercises (2h / week) / English

Mode of assessment
Written examination (120 min, 100%)

Requirement for the award of credit points
Passed final module examination

Module applicability
MSc. Computational Engineering

Weight of the mark for the final score
6 %

Module coordinator and lecturer(s)
Prof. Dr. P. Henning, Assistants

Further information